IDROSTATICA_A

Esercitazioni del corso di Fondamenti di idraulica

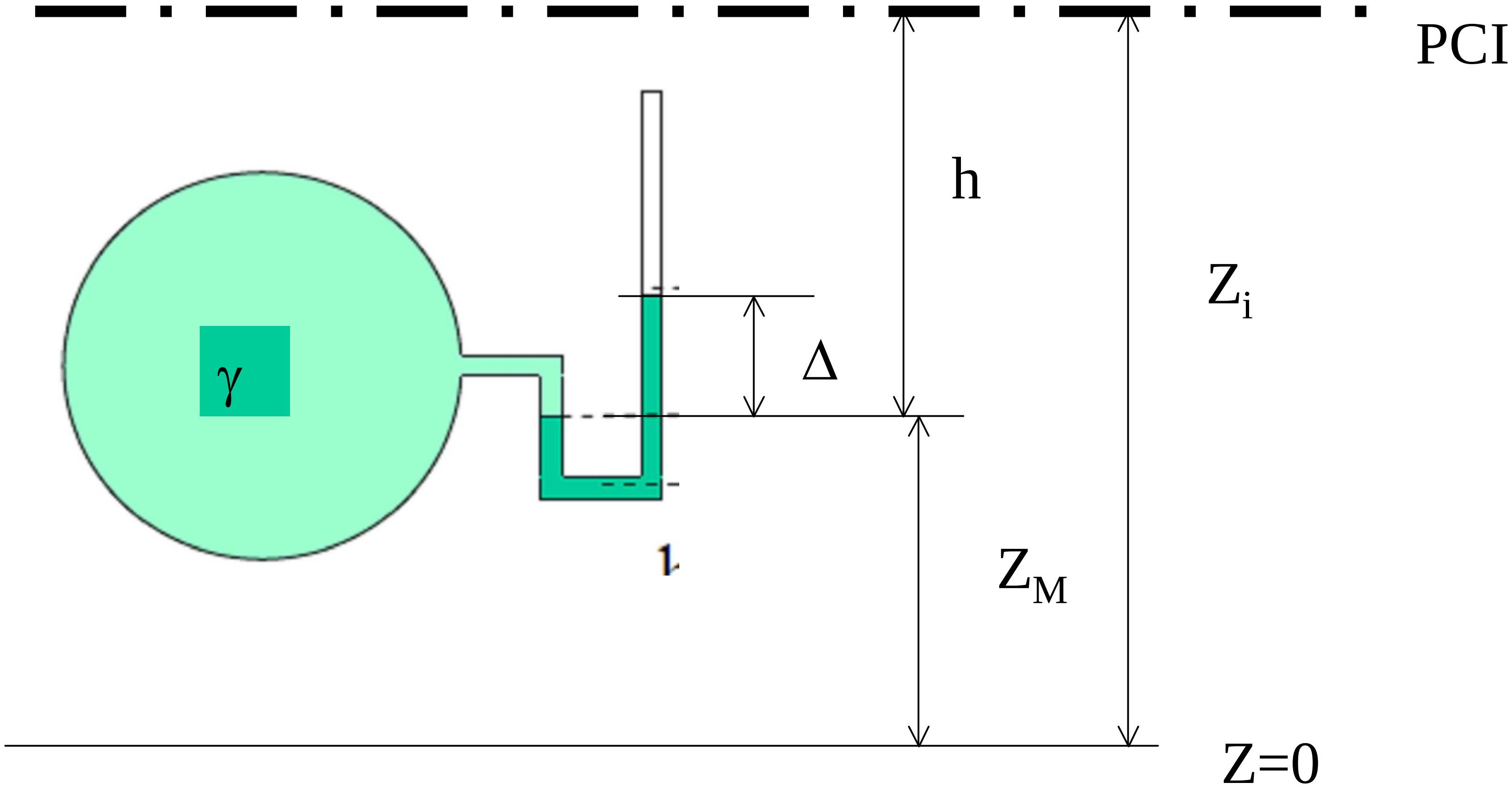
Dott.Ing. Gabriella Petaccia

petacciaunipv.it

Università degli Studi di Pavia

Dipartimento di Ingegneria Idraulica e Ambientale

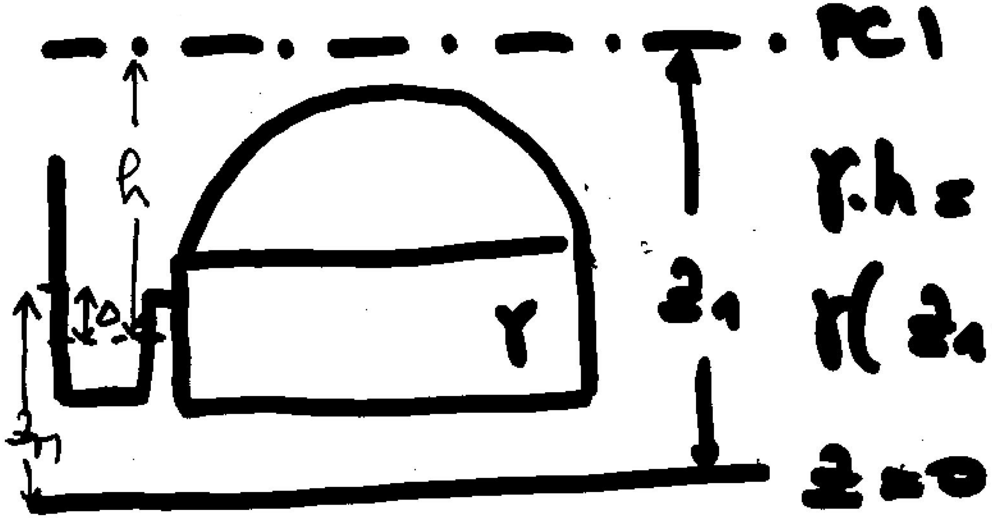
Misura delle pressioni: MANOMETRO SEMPLICE



Equilibrio pressioni sul piano passante sul menisco interno del manometro:

$$\Delta \cdot \gamma_M = \gamma \cdot h \quad \Rightarrow \quad h = \frac{\Delta \cdot \gamma_M}{\gamma} \quad \Rightarrow \quad z_i = h + z_M$$

Misura delle pressioni: MANOMETRO SEMPLICE



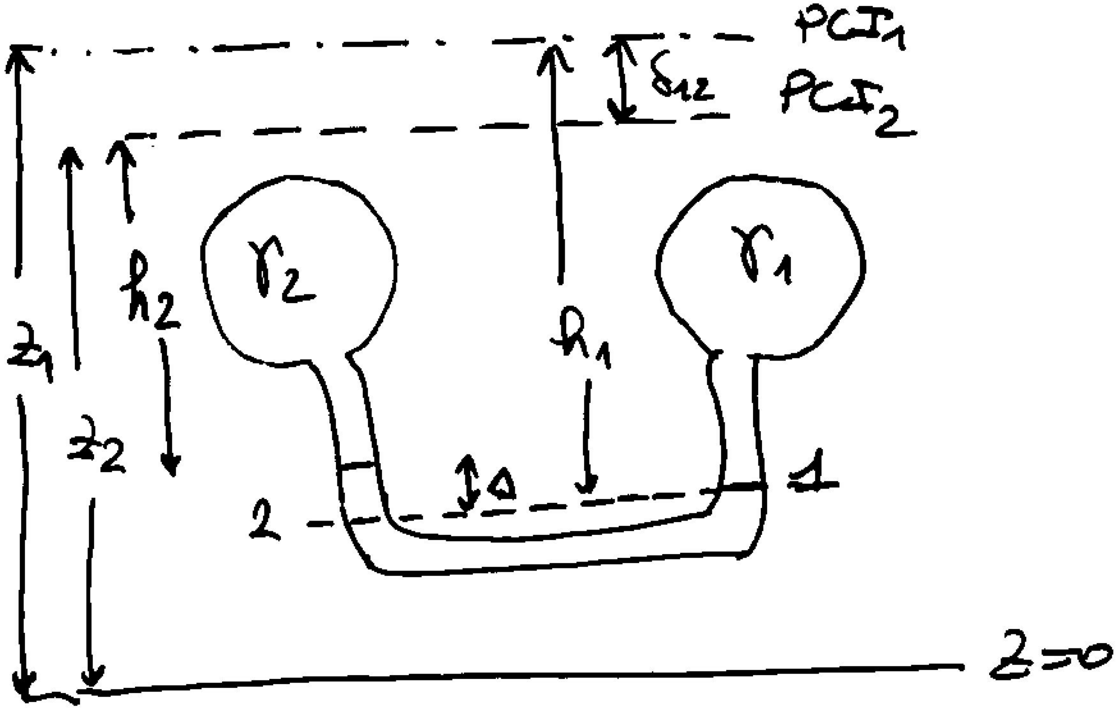
$$\gamma \cdot h = \Delta \gamma_m$$

$$\gamma(z_1 - z_m + \Delta) = \Delta \gamma_m$$

$$z_1 = z_m - \Delta + \Delta \frac{\gamma_m}{\gamma}$$

$$z_1 = z_m - \Delta\left(\frac{\gamma_m}{\gamma} - 1\right)$$

Misura delle pressioni: MANOMETRO DIFFERENZIALE



$\gamma_1 > \gamma_2$

$p_1 > p_2$

$$p_1 = p_2 + \Delta\,\gamma_M$$

$$p_1 = \gamma_1\,h_1$$

$$p_2 = \gamma_2 \cdot h_2$$

$$\gamma_1\,h_1 = \gamma_2\,h_2 + \Delta\,\gamma_M$$

$$h_1 = h_2 + \delta_{12} + \Delta$$

$$\gamma_1 (h_2 + \delta_{12} + \Delta) = \gamma_2 h_2 + \Delta \gamma_m$$

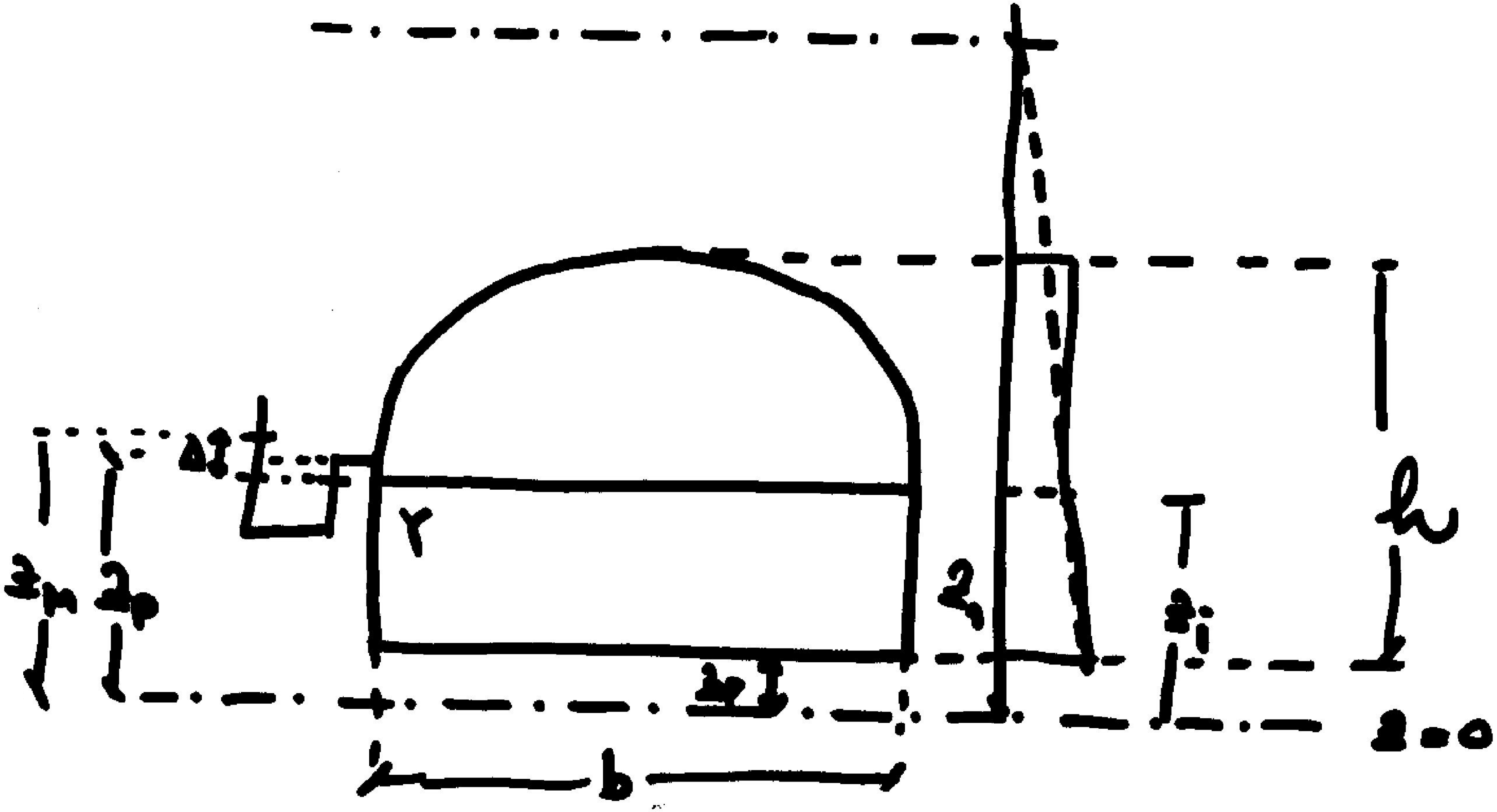
$$\gamma_1 h_2 + \gamma_1 \delta_{12} + \Delta \gamma_1 = \gamma_2 h_2 + \Delta \gamma_m$$

$$\delta_{12} = h_2 \frac{\gamma_2 - \gamma_1}{\gamma_1} + \Delta \frac{\gamma_m - \gamma_1}{\gamma_1}$$

CASO PARTICOLARE $\gamma_1 = \gamma_2 = \gamma$

$$\delta_{12} = h_2 \frac{\gamma - \gamma}{\gamma} + \Delta \frac{\gamma_m - \gamma}{\gamma} = \Delta \frac{\gamma_m - \gamma}{\gamma}$$

Idrostatica A – es1- manometro semplice



DATI

$b, h, \gamma, z_M, z_F, z_I, z_P, \gamma_M, \Delta, p_0$

SVOLGIMENTO

$z_P > z_I$

$$\gamma_M \cdot \Delta = \gamma (z_1 - z_I)$$

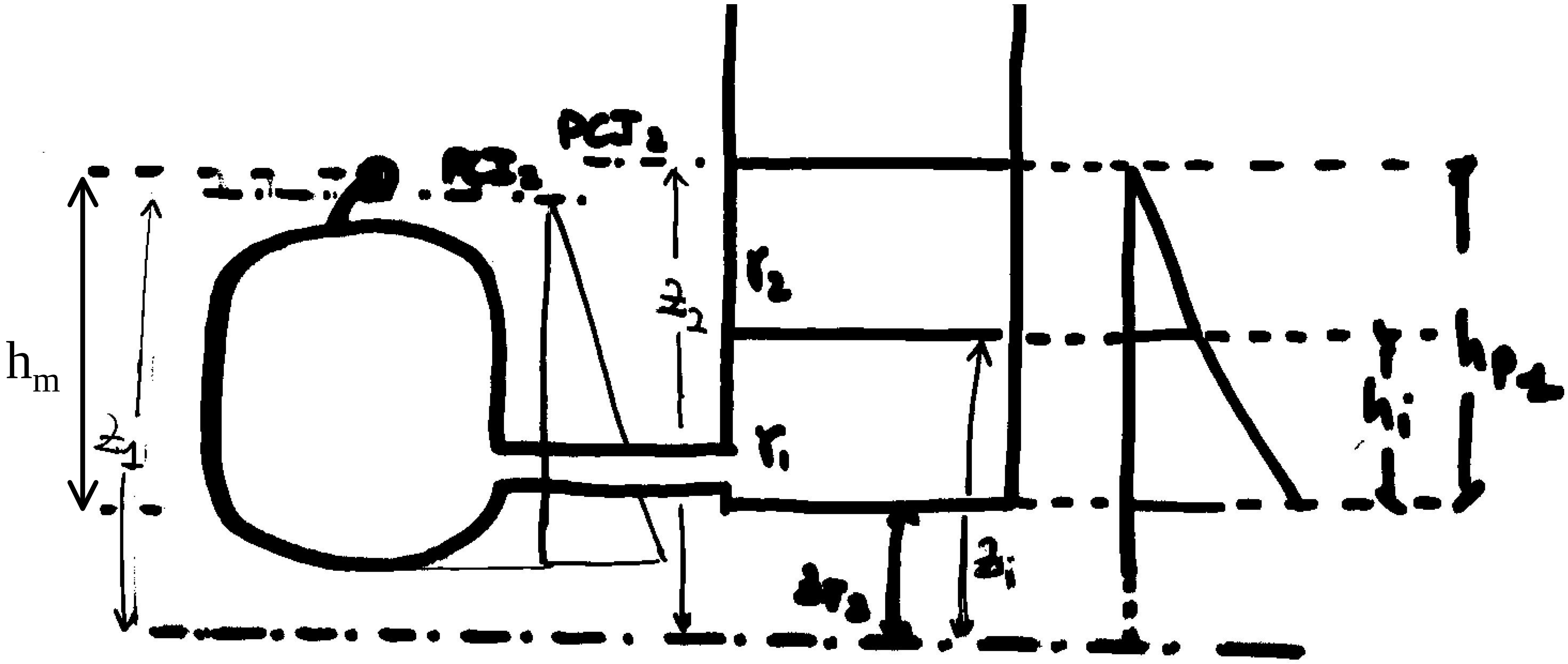
$$z_1 = \frac{\gamma_M}{\gamma} \cdot \Delta + z_I$$

$$p_I = \frac{\gamma (z_1 - z_I)}{1000} = p_a \quad [kPa]$$

$$p_a^*(kPa) = p_a + p_0$$

$$p_F = \frac{\gamma (z_1 - z_P)}{1000} \quad [kPa]$$

Idrostatica A – es2- manometro metallico



Idrostatica A – es2- manometro metallico

$$z_2 = h_{PL} + z_{F2}$$

$$z_I = h_I + z_{F2}$$

$$p_i\,(kPa) = \frac{\gamma_2 \cdot (z_2 - z_I)}{1000}$$

$$p_i = \frac{\gamma_1 (z_1 - z_I)}{1000} \Rightarrow z_1 = \frac{p_i \cdot 1000}{\gamma_1} + z_I$$

$$z_M = h_M + z_{F2}$$

$$M\,(kPa) = \frac{\gamma_1 \cdot (z_1 - z_M)}{1000}$$

Idrostatica A – es2- manometro metallico

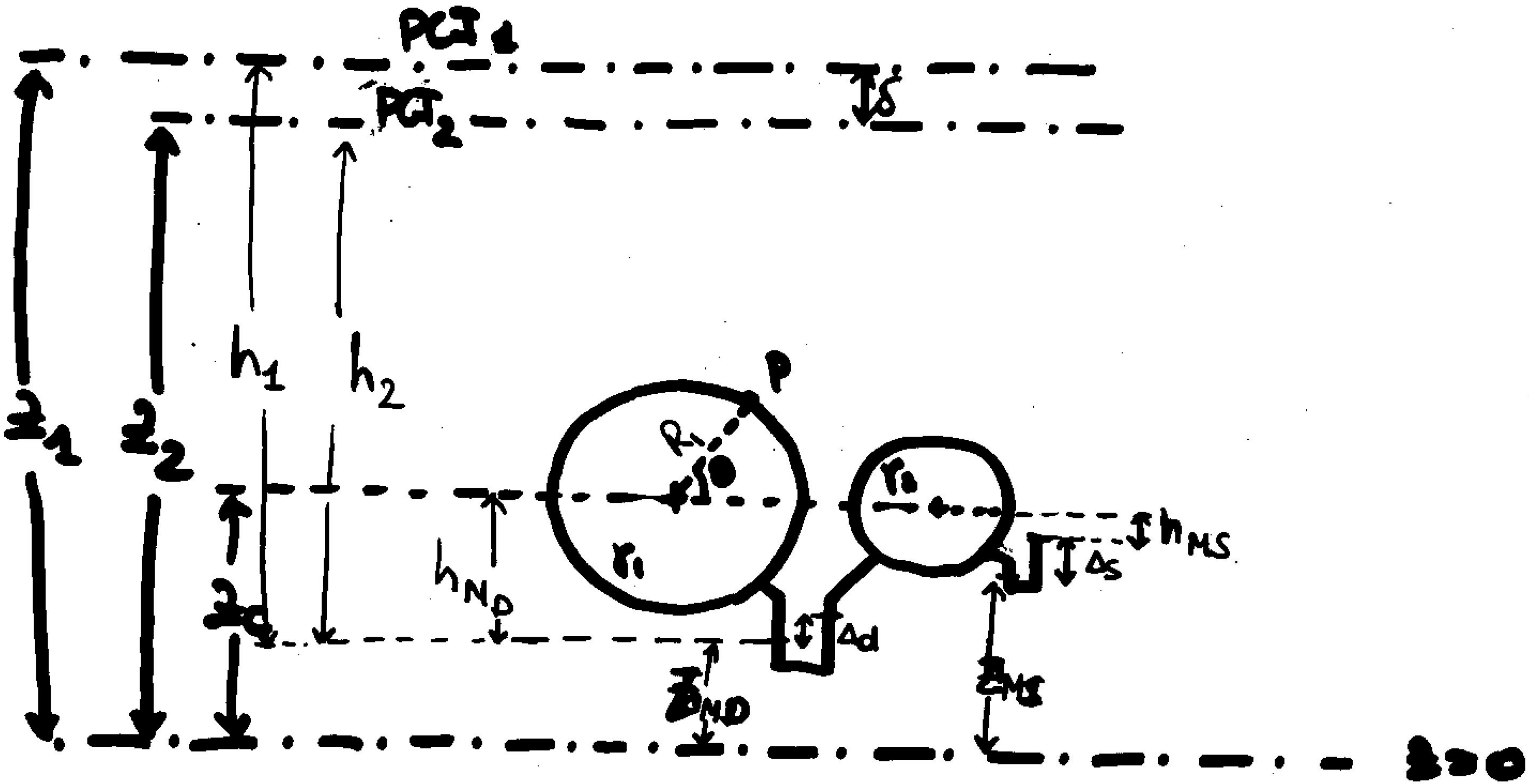
$$p^*(kPa) = AV + p_0$$

$$p_M^* = \gamma_1 \left(Z_1^* - Z_M \right)$$

$$\Rightarrow \quad Z_1^* = P_M^*/\gamma_1 \times 1000 + Z_M$$

$$\underset{(Pa)}{p_V} = \gamma_1 \left(Z_1^* - Z_L \right) \Rightarrow \quad Z_L = Z_1^* - \underset{(kPa)}{P_V}/\gamma_1 \times 1000$$

Idrostatica A – es3- manometro differenziale



Idrostatica A – es3- manometro differenziale

$$z_{MS} = z_C - h_{MS} - \Delta s$$

$$\Delta s \cdot \gamma_M = \gamma_2 (z_2 - z_{MS})$$

$$z_2 = \frac{\Delta s \cdot \gamma_M}{\gamma_2} + z_{MS}$$

$$z_{MD} = z_C - h_{MD}$$

$$h_2 = z_2 - z_{MD}$$

EQ. PRESSIONI MANOMETRO DIFFERENZIALE

$$h_1 \cdot \gamma_1 = \Delta d \cdot \gamma_M + \gamma_2 (h_2 - \Delta d)$$

$$h_1 = \frac{\Delta d \cdot \gamma_M + \gamma_2 (h_2 - \Delta d)}{\gamma_1} .$$

Idrostatica A – es3- manometro differenziale

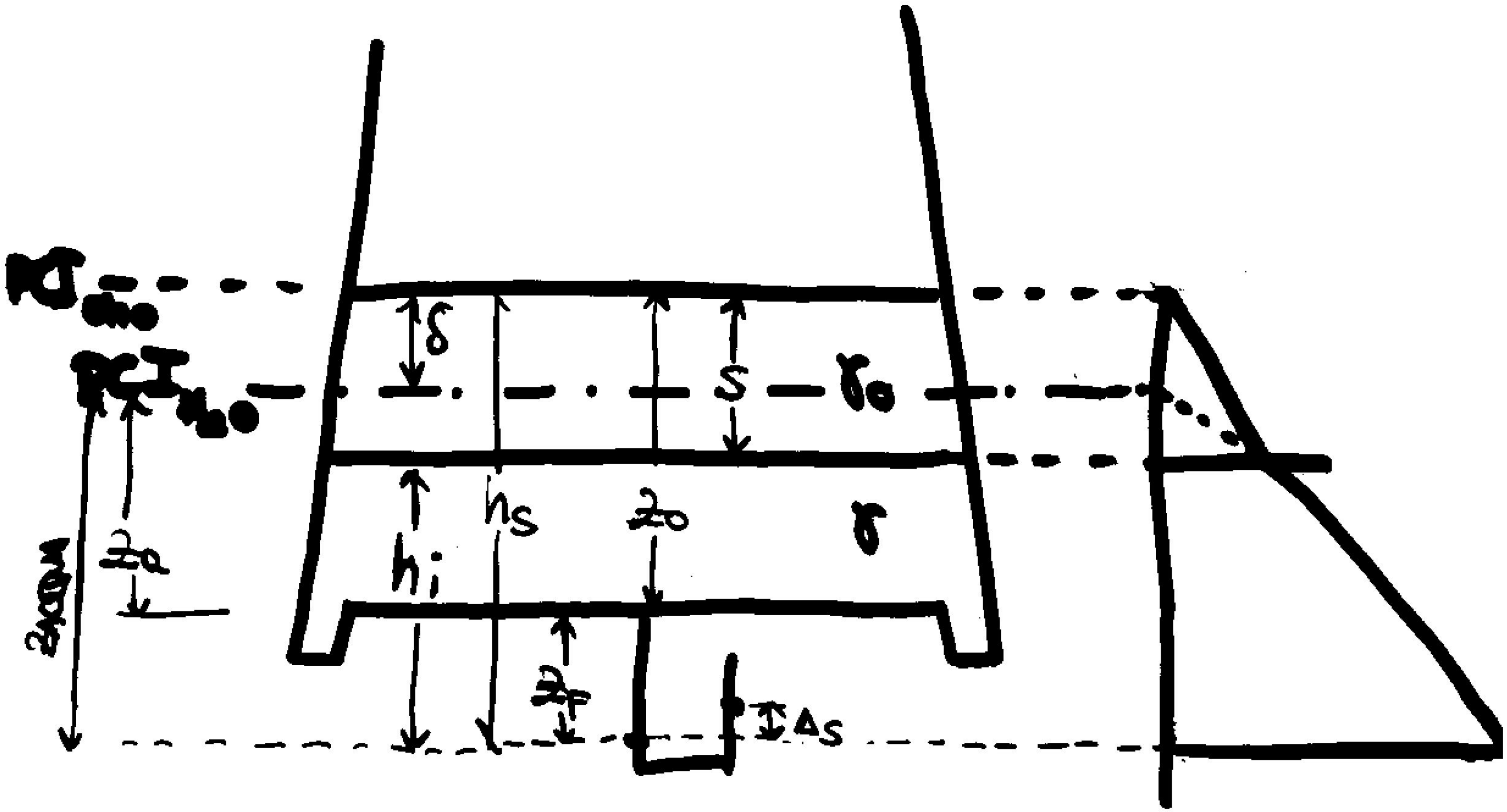
$$h_1 = \Delta d \frac{(\gamma_m - \gamma_2)}{\gamma_1} + \frac{\gamma_2}{\gamma_1} \cdot h_2$$

$$z_1 = h_1 + z_{NO}$$

$$z_p = z_c + R_1 \cdot \operatorname{sen}\theta$$

$$P_p(kPa) = \frac{\gamma_1 (z_1 - z_p)}{1000}$$

Idrostatica A – es4- misura delle pressioni



DATI

γ_0, γ, γ_M, z_0, Δs, h_s

Idrostatica A – es4- misura delle pressioni

$$p_S = \Delta_S \cdot \gamma_M$$

$$p_S = \gamma \cdot (h_S - \delta) \Rightarrow \delta = h_S - \frac{p_S}{\gamma}$$

Equilibrio pressioni all'interfaccia

$$s \cdot \gamma_0 = (s - \delta) \cdot \gamma$$

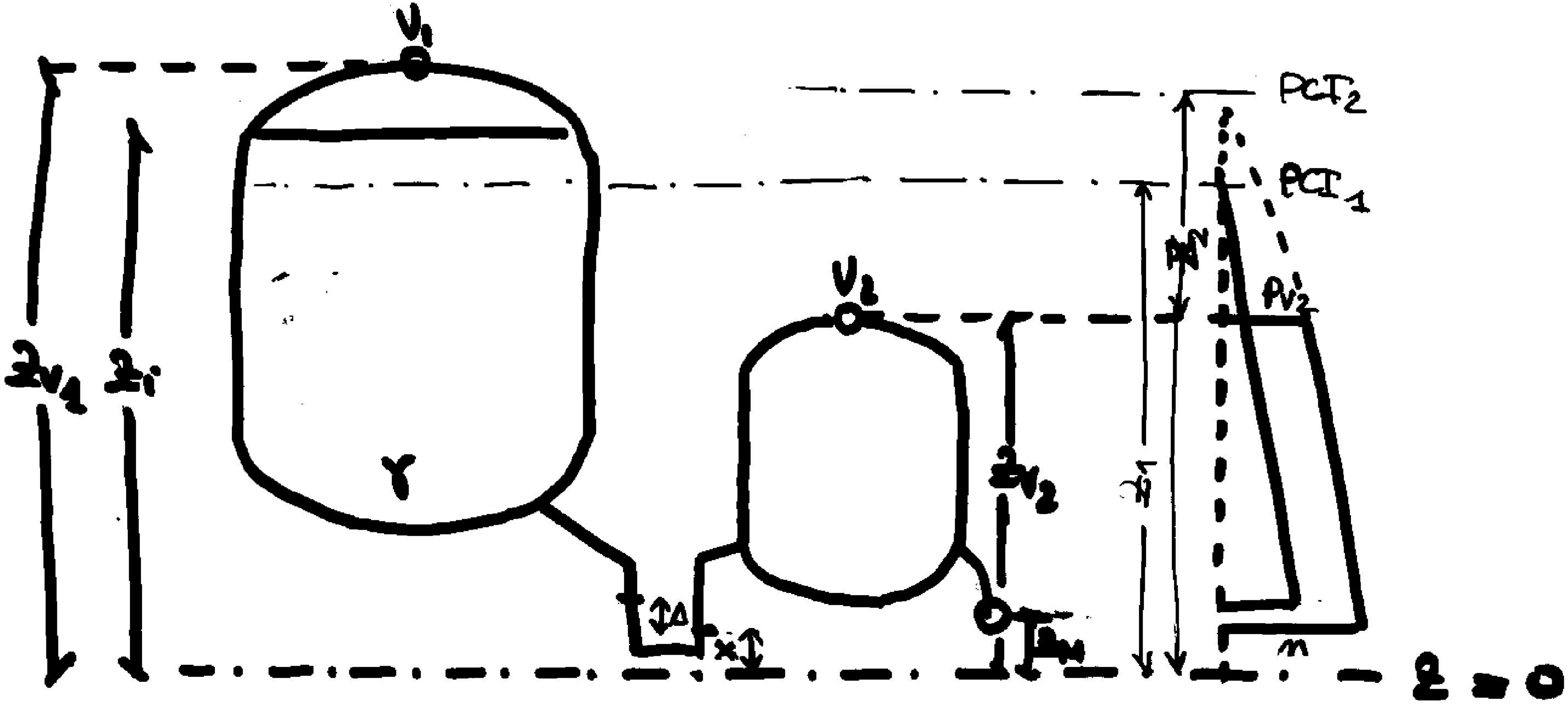
$$s \cdot \gamma_0 = s \cdot \gamma - \delta \cdot \gamma$$

$$s(\gamma_0 - \gamma) = -\delta \cdot \gamma$$

$$s = -\frac{\delta \cdot \gamma}{(\gamma_0 - \gamma)} = \frac{p_S}{\cancel{\gamma}} \frac{\cancel{\gamma}}{\gamma_0 - \gamma} - \frac{h_S \cdot \gamma}{\gamma_0 - \gamma} = \qquad h_i = h_S - s$$

$$= \frac{p_S - h_S \cdot \gamma}{\gamma_0 - \gamma} \qquad z_a = z_{ACQUA} - z_F = \frac{p_S}{\gamma} - (h_S - z_0)$$

Idrostatica A – es5- indicazioni manometriche



DATI: γ, γ_M, z_{V_1}, z_i, p_{V_1}; z_{V_2}; p_{V_2}; z_M

Idrostatica A – es5- indicazioni manometriche

$$m(kPa) = p_{V_2} + \frac{\gamma(z_{V_2} - z_m)}{1000}$$

Apertura valvola 2

$$m = \frac{\gamma(z_2 - z_m)}{1000} \Rightarrow$$

$$\frac{m \cdot 1000}{\gamma} = z_2 - z_m \Rightarrow z_2 = \frac{m \cdot 1000}{\gamma} + z_m$$

Apertura valvola 1

$$p_{V1} \cdot 1000 = \gamma(z_1 - z_i)$$

$$z_1 = \frac{p_{V1} \cdot 1000}{\gamma} + z_i$$

Idrostatica A – es5- indicazioni manometriche

MANOMETRO DIFFERENZIALE

$$\gamma(z_2 - x) = \Delta \cdot \gamma_m + \gamma(z_1 - \Delta - x)$$

$$z_2 \cdot \gamma - \cancel{\gamma \cdot x} = \Delta \cdot \gamma_m + \gamma \cdot z_1 - \Delta \cdot \gamma - \cancel{\gamma x}$$

$$z_2 \cdot \gamma = \Delta(\gamma_m - \gamma) + \gamma \cdot z_1$$

$$\Delta = \frac{\gamma(z_2 - z_1)}{\gamma_m - \gamma}$$

$$P_a = \gamma(z_A - z_i)/1000$$

SA/2

Idrostatica A – es6- lettura piezometrica

DATI

$\gamma_{H2O} = 9806 \frac{N}{m^3}$

$z_C = 3.80\ m$

$P_C = 30\ kPA$

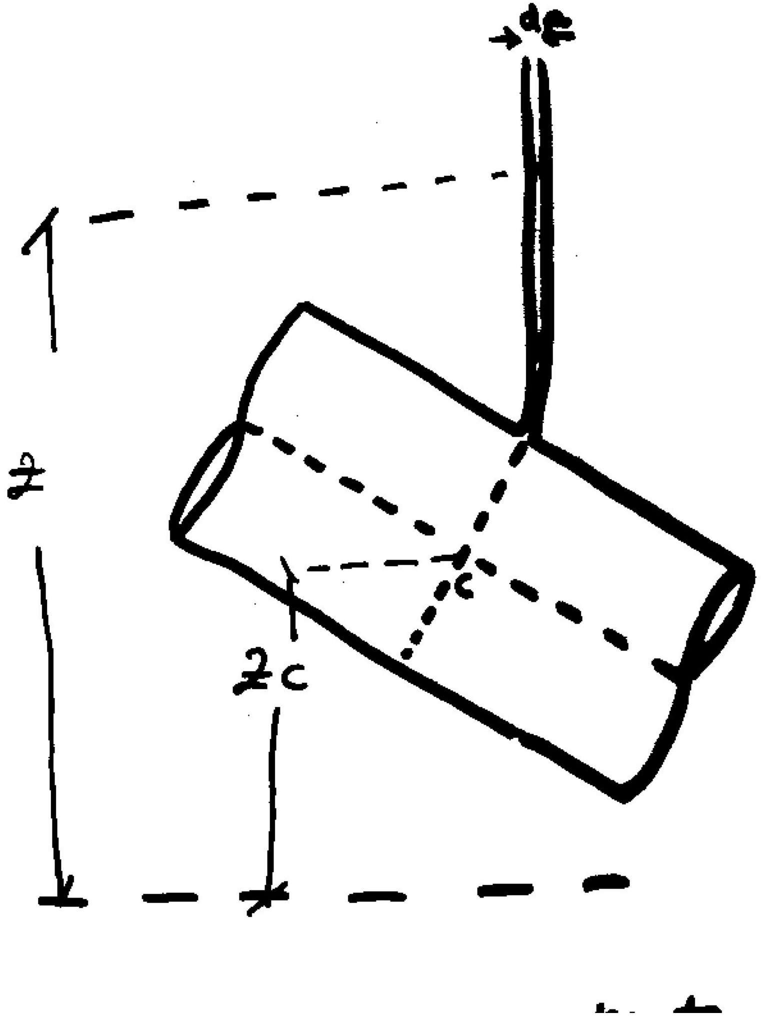
$d_m = 0.1\ m$

$S = 0.0725\ N/m$

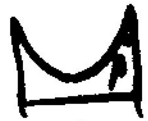
SVOLGIMENTO

In assenza di tensione superficiale il menisco si porterebbe alla quota!

$$z = z_C + \frac{P_C}{\gamma_{H2O}} \cdot 1000 = 6.86\ m$$



Per effetto della tensione superficiale l'acqua contenuta nel piezometro si solleva della quantità h:



d

$$S \cdot (2\pi r) \cdot \cos\beta = \gamma_{H2O} \cdot \pi r^2 h$$

Nel caso di acqua a contatto con parete di vetro

$\cos\beta = 1$

$$\Rightarrow h = \frac{S \cdot 2\pi r}{\pi r^2 \cdot \gamma_{H2O}} = \frac{2S}{r \cdot \gamma_{H2O}} = \frac{4S}{D \cdot \gamma_{H2O}} = 2.96 \cdot 10^{-4}\ m$$

La quota effettiva del menisco è allora

$$z_m = z + h \cong 6.86\ m$$

La pressione che erroneamente si rileverebbe da z_M GMi

$$p_E = \frac{\gamma_{H2O}\,(z_m - z_c)}{1000} = 30.00\ kPa$$

L'errore percentuale commesso nella misura sarebbe

$$e = \frac{P_E - P_C}{P_E} \cdot 100 = 0.02\ \%.$$